ACR No. L5F30

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

WARTIME REPORT

ORIGINALLY ISSUED

July 1945 as
Advance Confidential Report L5F30

EFFECT OF THE LIFT COEFFICIENT ON PROPELLER FLUTTER

By Theodore Theodorsen and Arthur A. Regier

Langley Memorial Aeronautical Laboratory
Langley Field, Va.

NACA

WASHINGTON

L - 161

NACA ACR No. L5F30

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

ADVANCE CONFIDENTIAL REPORT

EFFECT OF THE LIFT COEFFICIENT ON PROPELLER FLUTTER

By Theodore Theodorsen and Arthur A. Regier

SUMMARY

Flutter of propellers at high angles of attack is discussed, and flutter data obtained in connection with tests of models of large wind-tunnel propellers are analyzed and results presented. It is shown that in the high angle-of-attack range flutter of a propeller invariably occurs at a speed substantially below the classical flutter speed. The angle of attack at which flutter occurs appears to be nearly constant and independent of the initial blade setting. Thus, the blade simply twists to the critical position and flutter starts. Formulas have been developed which give an operating angle in terms of the design angle and other associated parameters, and these relations are presented in the form of graphs. It is seen that the flutter speed is lowered as the initial design lift coefficient is increased. It is further shown that by use of a proper camber of the propeller section the flutter speed may approach the classical value. A camber for which the blade will not twist is found to exist, and the corresponding lift coefficient is shown to be of special significance.

The stall (classical) flutter speed and the divergence speed of a propeller are shown to be approximately the same because of the centrifugal-force effects. It appears that a propeller will not flutter until the blade twists to a stall condition near the divergence speed. Stroboscopic observation of several propellers confirmed this theory. It was observed that, regardless of the initial pitch setting of the propeller, the blades always twisted to the stall condition before flutter commenced. The problem of predicting propeller flutter is thus resolved primarily into the calculation of the speed at which the propeller will stall.

INTRODUCTION

The present study of propeller flutter was conducted in connection with the design of several large wind-tunnel propellers for the Langley, Ames, and Cleveland Laboratories of the NACA. Wind-tunnel propellers are not required to operate in a fully stalled condition and can therefore be designed with small margin of safety against flutter. Airplane propellers, on the other hand, must have a considerable margin of safety since they are required to operate in the stall or near-stall condition in take-off. The results of the present tests are of wide interest since they apply to the general problem of the effect of high lift coefficients on the flutter velocity of a propeller.

There are two principal types of flutter: (1) "classical" flutter and (2) "stall" flutter. Classical flutter is an oscillatory instability of an airfoil operating in a potential flow. The problem of classical flutter was solved theoretically in reference 1. Stall flutter involves separation of the flow and occurs on airfoils operating near or in the stall condition of flow. Studer (reference 2) studied this type of flutter experimentally with an airfoil in two-dimensional flow. He found that the stall flutter speed was very much lower than the classical flutter speed and that, as the angle of attack of the airfoil was increased, the change from classical flutter to stall flutter was rather abrupt.

The problem of propeller flutter is somewhat different from the problem of wing flutter in that the change between classical flutter and stall flutter appears to be much more gradual. This gradual change of flutter speed with angle of attack has not been clearly understood, and attempts to calculate the flutter speed of propellers operating under normal loads have not been entirely satisfactory.

A third type of flutter, which may be referred to as "wake" flutter because it occurs on propellers operating at zero lift in their own wake, has sometimes been observed. Self-excited torsional oscillations of the propeller blade occur at frequencies which are integral multiples of the rotational speed of the propeller. At low speed the frequency of oscillation is equal to the torsional frequency of the propeller blade in still air.

The first such oscillation appears when the propeller rotational speed reaches approximately one-twentieth of the blade torsional frequency. This oscillation disappears as the rotational speed is increased but reappears at each integral multiple of the propeller speed until the classical flutter speed is reached. This type of flutter is not important for normal propeller operation.

SYMBOLS

L	representative length of propeller blade
c	chord of propeller section
b	semichord
t	thickness of propeller section
R	radius to tip of propeller
r	radius to propeller section
K	torsional stiffness of representative section
q	dynamic pressure of relative air stream
ρ	density
κ	ratio of mass of cylinder of air of diameter equal to chord of airfoil to mass of airfoil
α	angle of attack
$\Delta\alpha$	angle of twist or deformation of blade at representative section
α_{m_o}	angle of attack for which there is no twist
α_{l_o}	angle of attack for zero lift
$C_{m_{c/4}}$	moment coefficient about quarter-chord point
C_L	lift coefficient

C_{L_u} untwisted or design value of C_L

$C_{L_{u_I}}$ lift coefficient for ideal no-twist condition

x location of center of gravity as measured from leading edge

x_α location of center of gravity with reference to elastic axis in terms of semichord

a coordinate of torsional stiffness axis in terms of semichord as measured from midchord position

r_α nondimensional radius of gyration of airfoil section in terms of semichord referred to a

ω_α torsional frequency, radians per second

ω_h bending frequency, radians per second

M Mach number

v_D divergence speed

v_f flutter speed

v_{f_c} flutter speed corrected for compressibility

Subscripts:

u untwisted or design

cr critical

I ideal

c compressible

i incompressible

EFFECT OF LOADING ON THE TWIST OF A PROPELLER BLADE

The centrifugal force on a propeller has a component in the direction perpendicular to the relative flow, which is very nearly equal to the aerodynamic force. This statement is exactly true if the propeller is designed to avoid bending stresses and is approximately true in any case, since the bending forces are small compared with the aerodynamic forces. (See fig. 1.) The twist of the propeller at some representative section may be expressed by the relation

$$\Delta\alpha\ K = qLc^2\left(x - \frac{1}{4}\right)\frac{dC_L}{d\alpha}\left(\alpha_u + \Delta\alpha - \alpha_{m_o}\right) \qquad (1)$$

The value of the critical velocity q_{cr} for which divergence occurs is obtained from equation (1). Divergence evidently occurs for the condition

$$\Delta\alpha \rightarrow \infty$$

or

$$\frac{\Delta\alpha}{\alpha_u + \Delta\alpha - \alpha_{m_o}} \rightarrow 1$$

For $\Delta\alpha \rightarrow \infty$ then,

$$q_{cr} = \frac{K}{Lc^2\frac{dC_L}{d\alpha}\left(x - \frac{1}{4}\right)} \qquad (2)$$

By substitution of this value of q_{cr} for q in equation (1) the twist becomes

$$\Delta\alpha = \left(\alpha_u - \alpha_{m_o}\right)\frac{\frac{q}{q_{cr}}}{1 - \frac{q}{q_{cr}}} \qquad (3)$$

The moment coefficient around the quarter-chord point may be written as

$$C_{m_{c/4}} = \frac{dC_L}{d\alpha}\left(\alpha_{l_o} - \alpha_{m_o}\right)\left(x - \frac{1}{4}\right)$$

This equation may be rewritten as

$$\alpha_{m_o} = \alpha_{l_o} - \frac{C_{m_{c/4}}}{\left(x - \frac{1}{4}\right)\frac{dC_L}{d\alpha}} \tag{4}$$

The relation between the untwisted or design value of the lift coefficient C_{L_u} and the actual or measured value of C_L resulting from the twist may be expressed as

$$C_L = \frac{dC_L}{d\alpha}\left(\alpha_u - \alpha_{l_o} + \Delta\alpha\right)$$

or

$$C_L = C_{L_u} + \frac{dC_L}{d\alpha}\Delta\alpha \tag{5}$$

By substitution for $\Delta\alpha$ from equation (3) and by use of the value of α_{m_o} from equation (4) the following equation is obtained for C_L:

$$C_L = C_{L_u}\left(\frac{1}{1 - \frac{q}{q_{cr}}}\right) + C_{m_{c/4}}\frac{1}{x - \frac{1}{4}}\frac{\frac{q}{q_{cr}}}{1 - \frac{q}{q_{cr}}}$$

or the following equivalent relation is obtained:

$$C_{L_u} = C_L - \frac{q}{q_{cr}}\left(C_L + \frac{C_{m_{c/4}}}{x - \frac{1}{4}}\right) \tag{6}$$

The increase in lift coefficient due to twist is evidently

$$\Delta C_L = \frac{q}{q_{cr}}\left(C_L + \frac{C_{m_{c/4}}}{x - \frac{1}{4}}\right) \tag{7}$$

There is no increase in C_L, or no twist, if

$$C_{L_{u_I}} = -\frac{C_{m_{c/4}}}{x - \frac{1}{4}} \tag{8}$$

where $C_{L_{u_I}}$ indicates the value of C_L at which no twist occurs. This relation is plotted in figure 2. This figure shows that, for the Clark Y airfoil with center of gravity at 44 percent and $C_{m_{c/4}} = -0.07$, the value of the lift coefficient at which no twist of the blade is incurred is 0.37. In this case the angle of zero twist is not very far from the ideal angle of attack of the Clark Y airfoil, which is about 0.40. The following discussion shows that it is desirable to operate the propeller at the ideal angle of attack since operation at this angle delays stall and thus obviously causes an increase in the flutter speed. (See reference 3 for discussion of ideal angle of attack.)

A propeller, if generated as a true helix, will not be subjected to any centrifugal twisting moment; in fact, if the blade width of the propeller is adjusted to achieve the desired blade loading at an angle everywhere proportional to the helix angle, there will be no twist. This statement must be modified slightly, however, because the radial generating lines through the leading and trailing

[redacted]

edges may not be continued to the center and, as a result, the angle at the tip may be decreased. By proper plan form and distribution of mass, therefore, a propeller may be designed to have zero twist. A highly tapered propeller tends to decrease its tip angle due to centrifugal twisting moment, and a more nearly rectangular plan form induces an increase in the tip angle. These effects are in reality small compared with the indirect effect due to the aerodynamic forces resulting from the bending of the blade.

EXPERIMENTAL STUDIES OF FLUTTER OF PROPELLERS AT HIGH LOADINGS

A number of propellers of different designs, some representing existing NACA wind-tunnel propellers for which data were available and others representing proposed wind-tunnel propellers, were tested as wind-tunnel fans in a small open tunnel. A cross-sectional sketch of the test setup is shown in figure 3.

The lift coefficient of the blades was changed by changing the area of the tunnel exit. The value of C_{L_u} was calculated from the relative wind direction at the 0.8-radius station and the angle of attack of the untwisted blade.

The propeller tips were observed by stroboscope through a small window in the tunnel wall, centered in the plane of the propeller. Blade-tip deflection and twist and the pressure increase of the air passing through the fan were recorded. These data were used to give an independent check on the operating lift coefficient. Owing to the nonuniformity of the blades, the variation of the lift coefficient with the radius, and other causes, the value of the operating lift coefficient could be determined only within about 0.1 to 0.2.

Although a number of propellers were tested, only results from the tests of two fairly representative propellers are reported. The propellers were made of laminated spruce and had flat-bottom Clark Y sections. Propeller A, for which data are given in figure 4, was a six-blade propeller 45 inches in diameter. Propeller B was a single-blade propeller having the same diameter but

two-thirds the chord and thickness of propeller A. The propeller of smaller cross section, propeller B, was used to reduce the flutter speed and to make possible a study of the flutter modes. Propeller B was tested in the same location in the tunnel as the other propellers, but a booster fan was attached at the rear of the motor to force the air through the tunnel during the tests of propeller B.

The vibration frequencies of the propellers are as follows:

Mode	Vibration frequency (cps)	
	Propeller A	Propeller B
First bending	74	44
Second bending	246	172
Torsion	355	340

Torsion and bending strain gages were attached to propeller B, and the flutter amplitudes and frequencies were recorded. The results of these tests are given in figure 5. The flutter speed was changed by changing the blade lift coefficient. It may be observed that the bending amplitude is large and the flutter frequency is low (170 cps) at the highest observed flutter speed. In a lower range of flutter speed - from 400 to 500 feet per second - the observed bending amplitude is small and the flutter frequency attains a higher value, 240 cycles per second instead of 170. This flutter evidently involves the second bending mode, whereas the flutter at top speed involves the first bending mode. At the lowest flutter speed and the highest blade loading, the flutter reverts to a condition of pure torsional oscillation at a frequency corresponding to that of pure torsion measured in free air, namely, 340 cycles per second.

The results of the flutter tests of propeller A are shown in figure 6, which is a plot of equation (6) for $C_{L_{u_I}} = 0.37$, the value of $C_{L_{u_I}}$ for a Clark Y flat-bottom airfoil of 12-percent thickness. The straight lines that converge at $\frac{q}{q_{cr}} = 1.0$ and $C_{L_{u_I}} = 0.37$ in

figure 6 are lines representing constant values of C_L. Figure 6 shows that, if the design lift coefficient C_{L_u} is 0.6, for instance, the blade will twist so that C_L is 1.0 at $\frac{q}{q_{cr}} = 0.63$. Diagrams similar to figure 6 may be used to obtain C_L for any propeller in terms of C_{L_u}.

Figure 6 also shows curves for data plotted with C_{L_u} uncorrected for compressibility and for the same data plotted with C_{L_u} corrected for compressibility by Glauert's formula

$$C_{L_{u_c}} = \frac{C_{L_{u_i}}}{\sqrt{1 - M^2}}$$

The data corrected for compressibility show that, when the propeller was set with the blade at stall, the flutter speed q/q_{cr} was only 0.17. As C_{L_u} was decreased from 0.85 to 0.65, q/q_{cr} increased from 0.37 to 0.62. It may be noted that in this range the flutter occurred at an approximately constant C_L of 1.1. A further increase in q/q_{cr} caused a rather sharp drop in C_L for flutter. The data uncorrected for compressibility show that, as C_{L_u} varies from 0.78 to 0.55, C_L for flutter varies from 1.1 to 0.82. The validity of the compressibility correction as applied to C_L in figure 6 may be questioned, but results of the tests clearly indicate that the propeller twisted to a lift coefficient near unity before flutter occurred. The propeller can carry a lift coefficient exceeding unity without flutter if q/q_{cr} is low enough; however, as the classical flutter speed is approached, the flutter lift coefficient becomes less than unity. In other words, the amount of stall necessary to excite flutter is less as the classical flutter speed is approached. This conclusion is in agreement with the experiment of reference 2 and seems logical in view of the various flutter modes described by figure 5.

The minimum flutter speed for a completely stalled propeller is of importance for propellers that operate in such a condition at times - in take-off, for example.

[redacted]

There is no reliable method known for calculating this minimum flutter speed for a completely stalled propeller, and further investigation of the problem is required. As the angle of attack is increased from a normal value, the flutter speed will decrease until a minimum is reached. There is evidence that this minimum is related to the von Kármán vortex street; the minimum probably corresponds to a coincidence of the torsional frequency with that of the von Kármán vortex street. In figure 6 it may be observed that this minimum appears at $\frac{q}{q_{cr}} = 0.17$ for propeller A. This value may be fair for wooden propellers but cannot be taken as valid for metal propellers. Flutter on metal propellers operating in the stall condition has been observed to occur at values of q/q_{cr} as low as 0.04.

Figure 7 is of interest as a verification of the theoretical treatment in this paper. It gives the twist of the propeller tip as a function of C_{L_u} for a constant propeller speed at $\frac{q}{q_{cr}} = 0.37$. This twist was observed by means of telescope and stroboscope. The line on the figure is drawn through the point for an angle of twist of 0° and a value of C_{L_u} of 0.37, as predicted by figure 2. The slope of this line, which may be calculated by use of equation (3), is adjusted to fit the data. Equation (3) is based on a simplified propeller and gives the twist at the representative section. At $C_{L_u} = 0.78$ and $\frac{q}{q_{cr}} = 0.37$, equation (3) gives $\Delta\alpha = 2.4^{\circ}$. The observed twist at the propeller tip for this condition was 5.1°. It was observed by direct measurement that the torsional stiffness at the representative section was 2.3 times that at the tip. The observed tip twist is therefore consistent with the expected value.

DETERMINATION OF DIVERGENCE SPEED AND CLASSICAL FLUTTER SPEED

The divergence speed of a propeller can be calculated from equation (2) if proper values are selected for L, c, and K but may be more conveniently found from the formula of reference 4 (p. 17) which is

$$\frac{v_D}{b\omega_\alpha} = \sqrt{\frac{r_\alpha^{\,2}}{\kappa} \frac{\frac{1}{2}}{\frac{1}{2} + a}} \tag{9}$$

Reference 4 (p. 17) also gives an approximate flutter formula that appears to hold very well for a heavy wing and for small values of ω_h/ω_α - the conditions for normal propellers. The approximate flutter formula is

$$\frac{v_f}{b\omega_\alpha} \approx \sqrt{\frac{r_\alpha^{\,2}}{\kappa} \frac{\frac{1}{2}}{\frac{1}{2} + a + x_\alpha}} \tag{10}$$

where x_α is the location of the center of gravity with reference to the elastic axis in terms of the semichord. It may be noted that formulas (9) and (10) are alike except for an additional term x_α in formula (10). If the center-of-gravity location coincides with the location of the elastic axis, $x_\alpha = 0$, the two formulas are identical.

It was shown in the discussion of figure 1 that there are two moments acting about the elastic axis, the aerodynamic-force moment and the centrifugal-force moment. If the aerodynamic forces are balanced by components of the centrifugal force, the resulting moment is the same as if the aerodynamic forces acted with a moment taken about the center of gravity of the airfoil section. For propellers, therefore, the dynamic-stiffness axis may be taken at the center of gravity and the divergence speed of the propeller will be given approximately by the flutter formula, equation (10), or $v_D \approx v_f$. It may be mentioned that for normal propellers the location of the elastic axis and the location of the center of gravity are usually very close together.

The location of the center of gravity from equation (10), $a + x_\alpha$, is expressed in terms of the semichord as measured from the midchord position. Equation (10) may be written

$$\frac{v_f}{b\omega_\alpha} \approx \sqrt{\frac{r_\alpha^2}{\kappa}\frac{\frac{1}{4}}{x - \frac{1}{4}}} \tag{11}$$

where x is the location of the center of gravity in fraction of chord as measured from the leading edge. Since $v_D \approx v_f$,

$$q_{cr} = \frac{1}{2}\rho v_f^2$$

Propellers usually operate near a Mach number of one, and the compressibility correction therefore becomes extremely important. As yet, there is no accurate knowledge concerning the compressibility correction for the flutter velocity near the velocity of sound. An approximate compressibility correction for the subsonic range from reference 4 is

$$M_c^2 \approx M_i^2\left(1 - \frac{M_i^2}{2} + \frac{M_i^4}{8} - . . .\right)$$

where M_c is the Mach number corresponding to flutter speed in compressible flow and M_i is the Mach number corresponding to flutter speed in incompressible flow. The flutter speed corrected for compressibility is tentatively calculated in the appendix and is indicated in figure 6 by the vertical line at $\frac{q}{q_{cr}} = 0.79$.

The choice of the radius of the representative section is open to some question. Since the velocity varies approximately as the radius, this choice is rather important. It has been customary to use the section at three-fourths semispan as the representative section for wings. Because of the velocity distribution on propellers, the representative section was taken at the 0.8-radius station.

CONCLUDING REMARKS

It has been shown that the stall flutter speed of a propeller is in general very much lower than the calculated classical flutter speed.

The classical flutter speed may be attained only if the propeller operates at the ideal angle of zero twist. The ideal angle of zero twist depends on the moment coefficient of the section, and the corresponding lift coefficient has been given by a simple relation.

It is desirable to have the design angle equal to the ideal angle of attack in order that the speed at which flutter occurs may be higher. The design angle should therefore be equal both to the ideal angle and to the ideal angle of zero twist.

Langley Memorial Aeronautical Laboratory
National Advisory Committee for Aeronautics
Langley Field, Va.

APPENDIX

SAMPLE CALCULATION OF FLUTTER SPEED CORRECTED FOR COMPRESSIBILITY FOR PROPELLER A

Propeller section characteristics at 0.8-radius station:

Type - Clark Y flat-bottom 12-percent-thick airfoil

$$x = 0.44$$

$$r_\alpha^2 = 0.24$$

Specific gravity = 0.5

$$\kappa = \frac{1}{45}$$

$$\frac{c}{R} = 0.098$$

Propeller characteristics:

$$R = 1.87 \text{ ft}$$

$$\omega_\alpha = 2\pi\ (355)$$

$$b = \frac{c}{2} = \frac{R}{2}\frac{c}{R} = \left(\frac{1.87}{2}\right)(0.098) = 0.092$$

$$v_f \approx b\omega_\alpha\sqrt{\frac{r_\alpha^2}{\kappa}\,\frac{\frac{1}{4}}{x - \frac{1}{4}}}$$

$$\approx (0.092)(2\pi)(355)\sqrt{\frac{(0.24)(45)(0.25)}{0.44 - 0.25}} = 772 \text{ fps}$$

[redacted]

Correction for compressibility:

$$M_1 = \frac{772}{1120} = 0.69$$

$$M_c = \sqrt{M_1^2\left(1 - \frac{1}{2}M_1^2 + \frac{1}{8}M_1^4\right)} = 0.612$$

$$v_{f_c} = (0.612)(1120) = 685 \text{ fps}$$

$$\left(\frac{q}{q_{cr}}\right)_c = \left(\frac{v_{f_c}}{v_f}\right)^2 = \left(\frac{685}{772}\right)^2 = 0.79$$

This value represents the flutter speed corrected for compressibility. (See fig. 6.)

REFERENCES

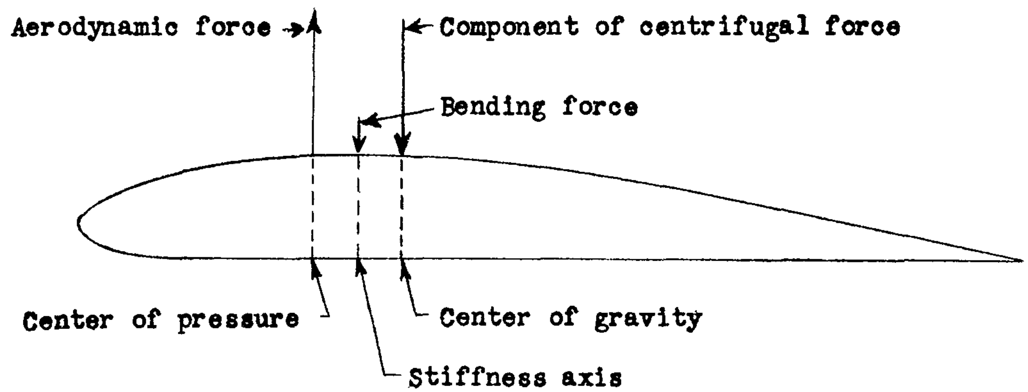
1. Theodorsen, Theodore: General Theory of Aerodynamic Instability and the Mechanism of Flutter. NACA Rep. No. 496, 1935.

2. Studer, Hans-Luzi: Experimentelle Untersuchungen über Flügelschwingungen. Mitteilung no. 4, Inst. Aerod. Tech. H. S. Zurich, Gebr. Leemann & Co. (Zurich), 1936.

3. Theodorsen, Theodore: On the Theory of Wing Sections with Particular Reference to the Lift Distribution. NACA Rep. No. 383, 1931.

4. Theodorsen, Theodore, and Garrick, I. E.: Mechanism of Flutter - A Theoretical and Experimental Investigation of the Flutter Problem. NACA Rep. No. 685, 1940.

[redacted]



[redacted]

NATIONAL ADVISORY
COMMITTEE FOR AERONAUTICS

Figure 1.- Forces on a representative propeller section.

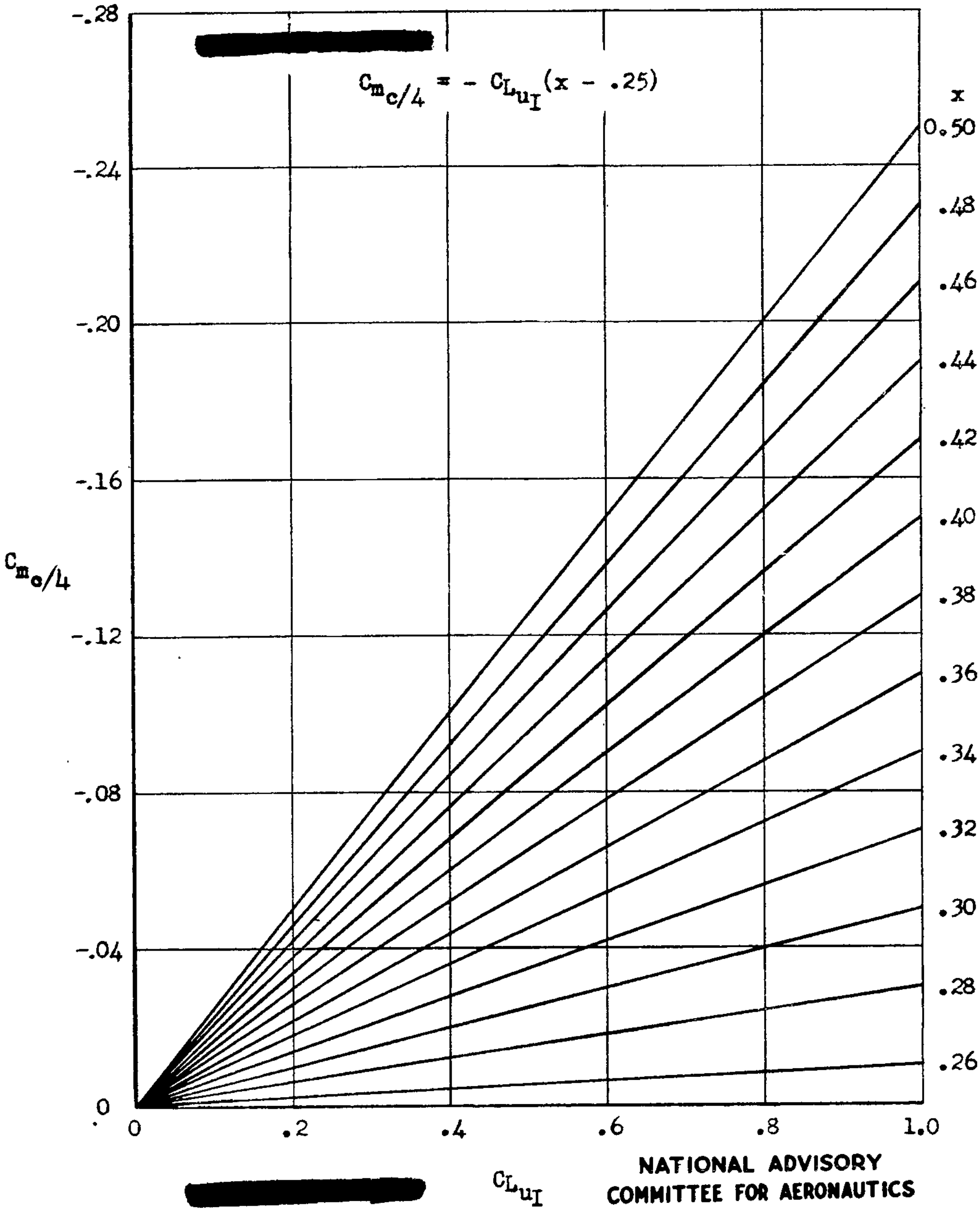
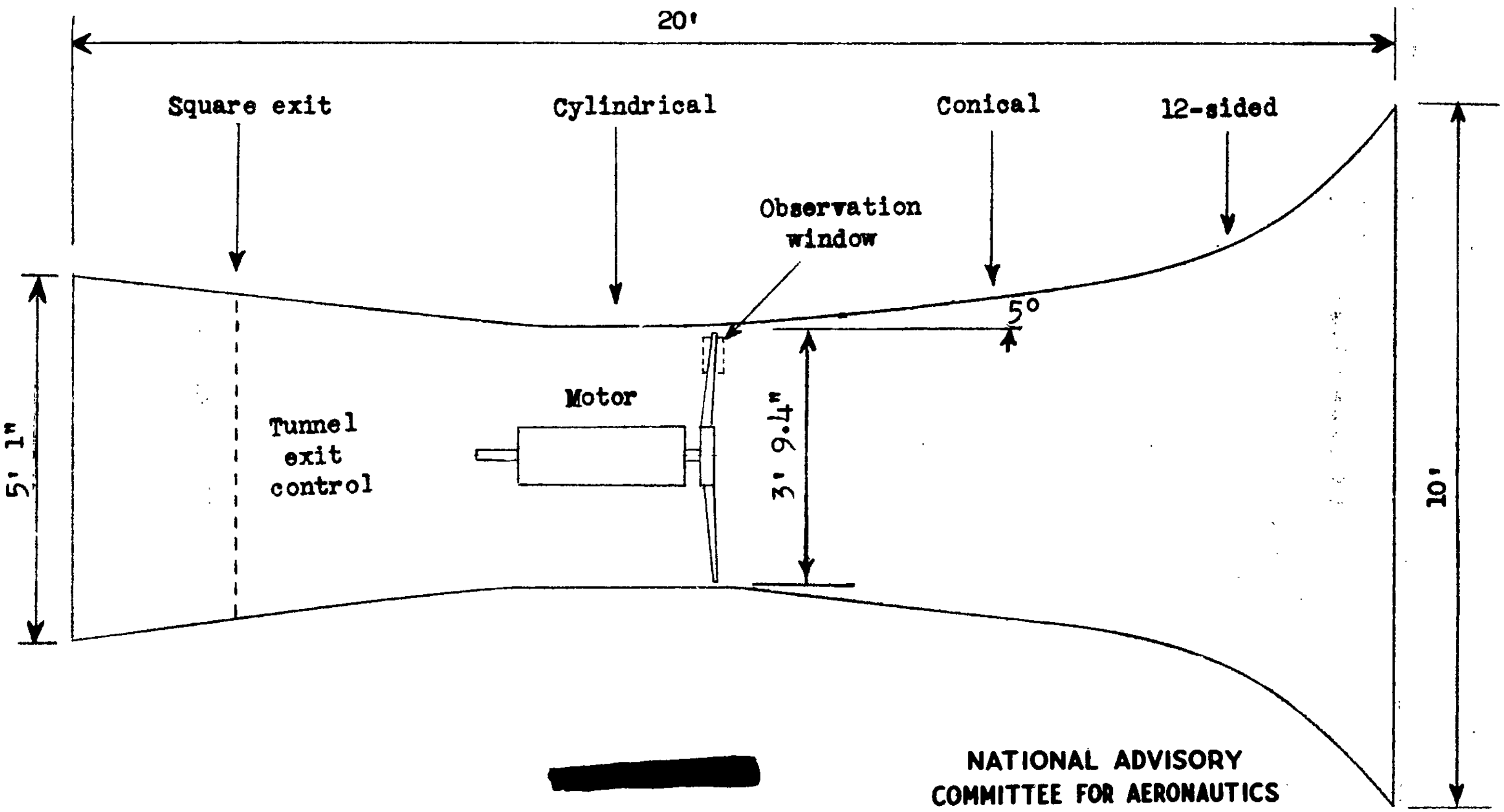


Figure 2.- Moment coefficient $C_{m_{c/4}}$ plotted as function of lift coefficient $C_{L_{u_I}}$ for various values of center-of-gravity location x.

[redacted]



[redacted]

Figure 3.- Cross-sectional view of wind tunnel used for tests.

[redacted]

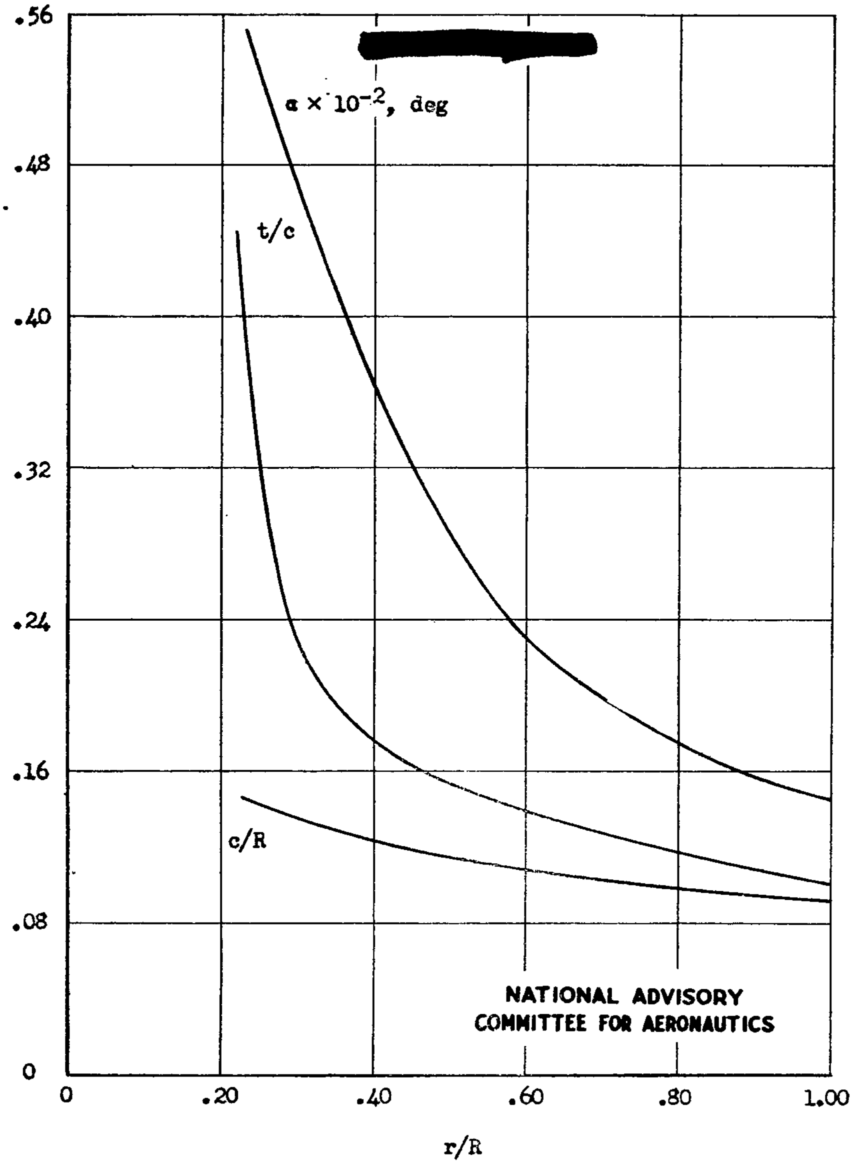


Figure 4.- Data for propeller A.

[redacted]

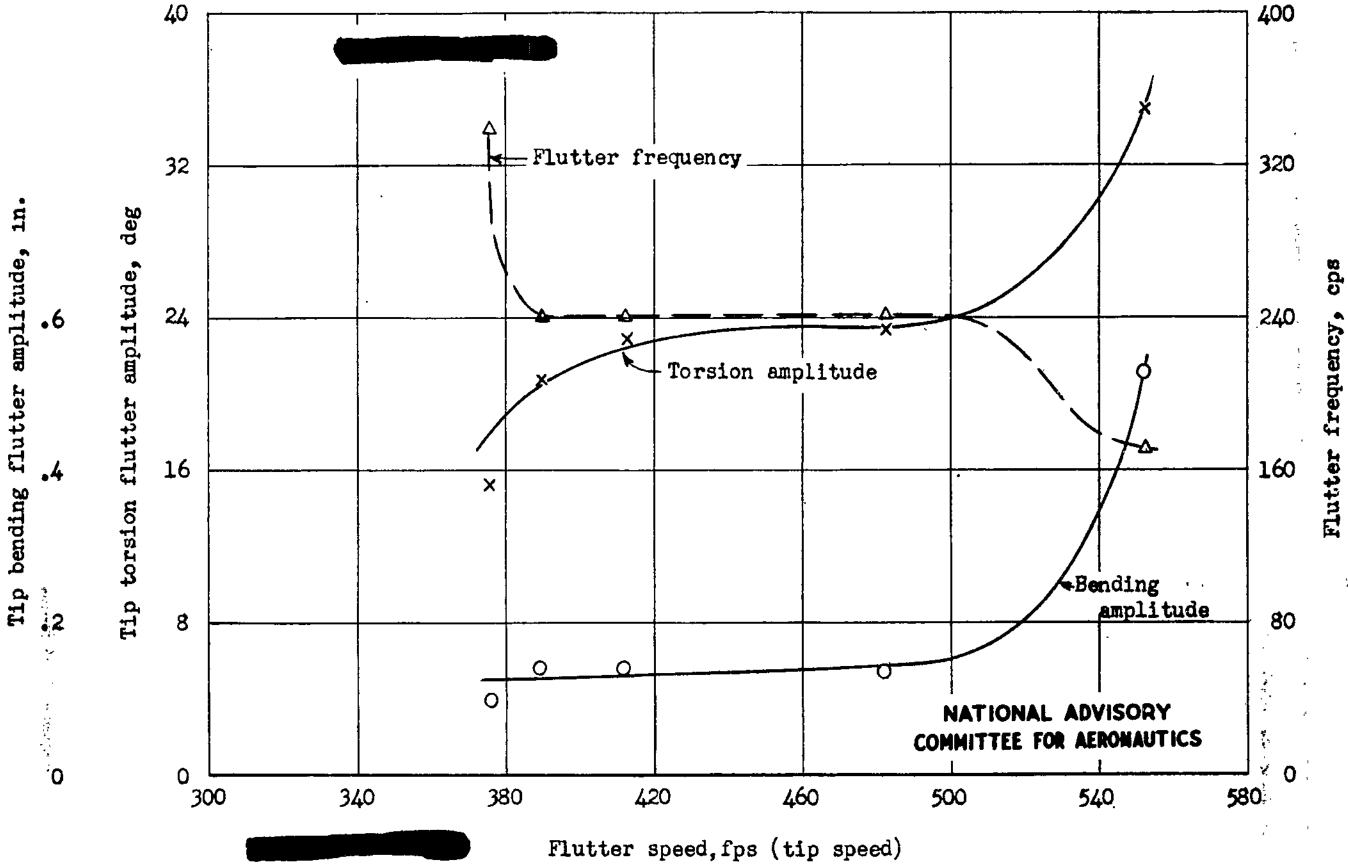
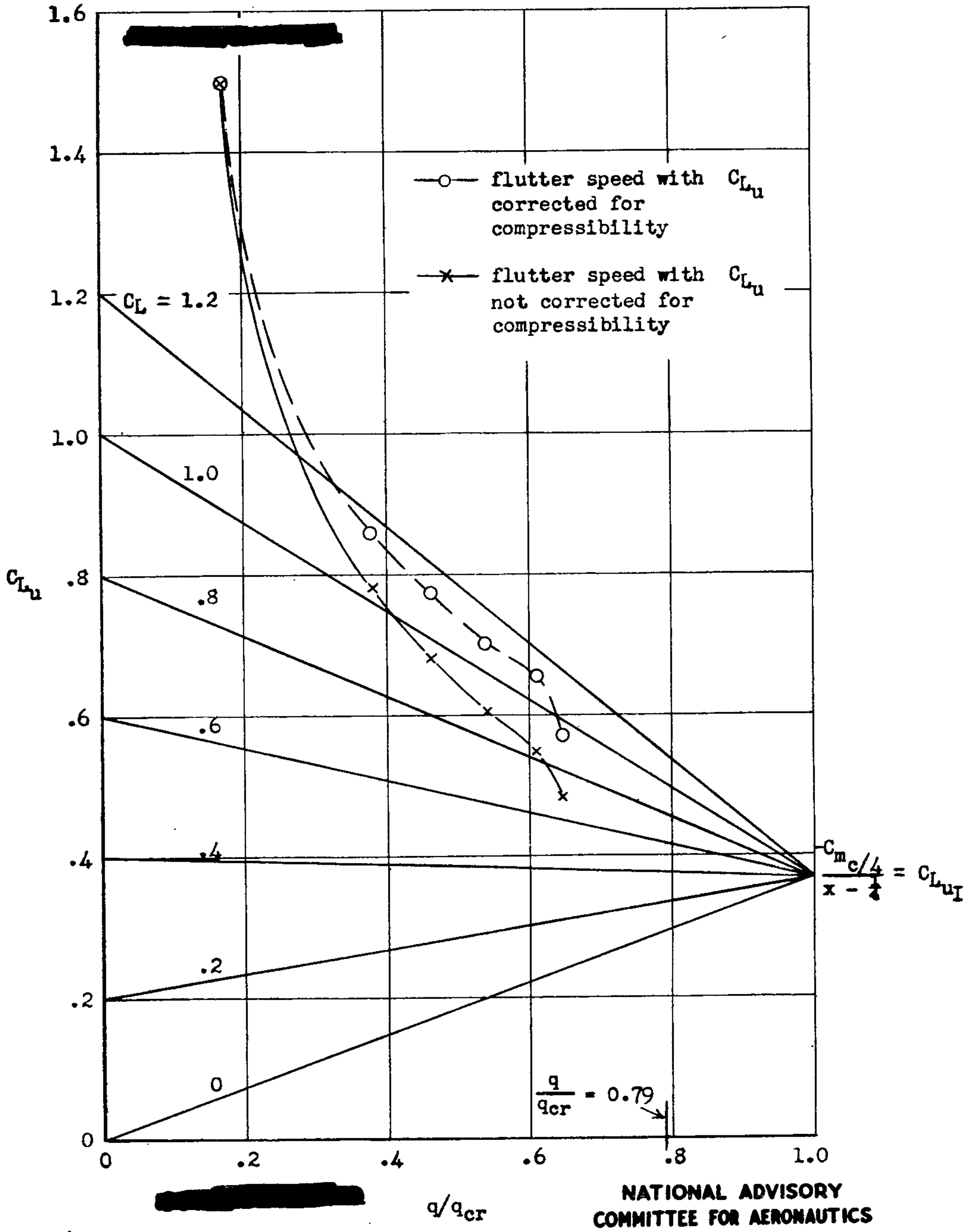


Figure 5.- Bending and torsion amplitudes and flutter frequency of propeller B as function of flutter speed. Torsional frequency in still air, 340 cycles per second.



NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

Figure 6.- Flutter speed q/q_{cr} as function of C_{L_u} for propeller A. $q/q_{cr} = 0.79$ is the calculated flutter speed corrected for compressibility.

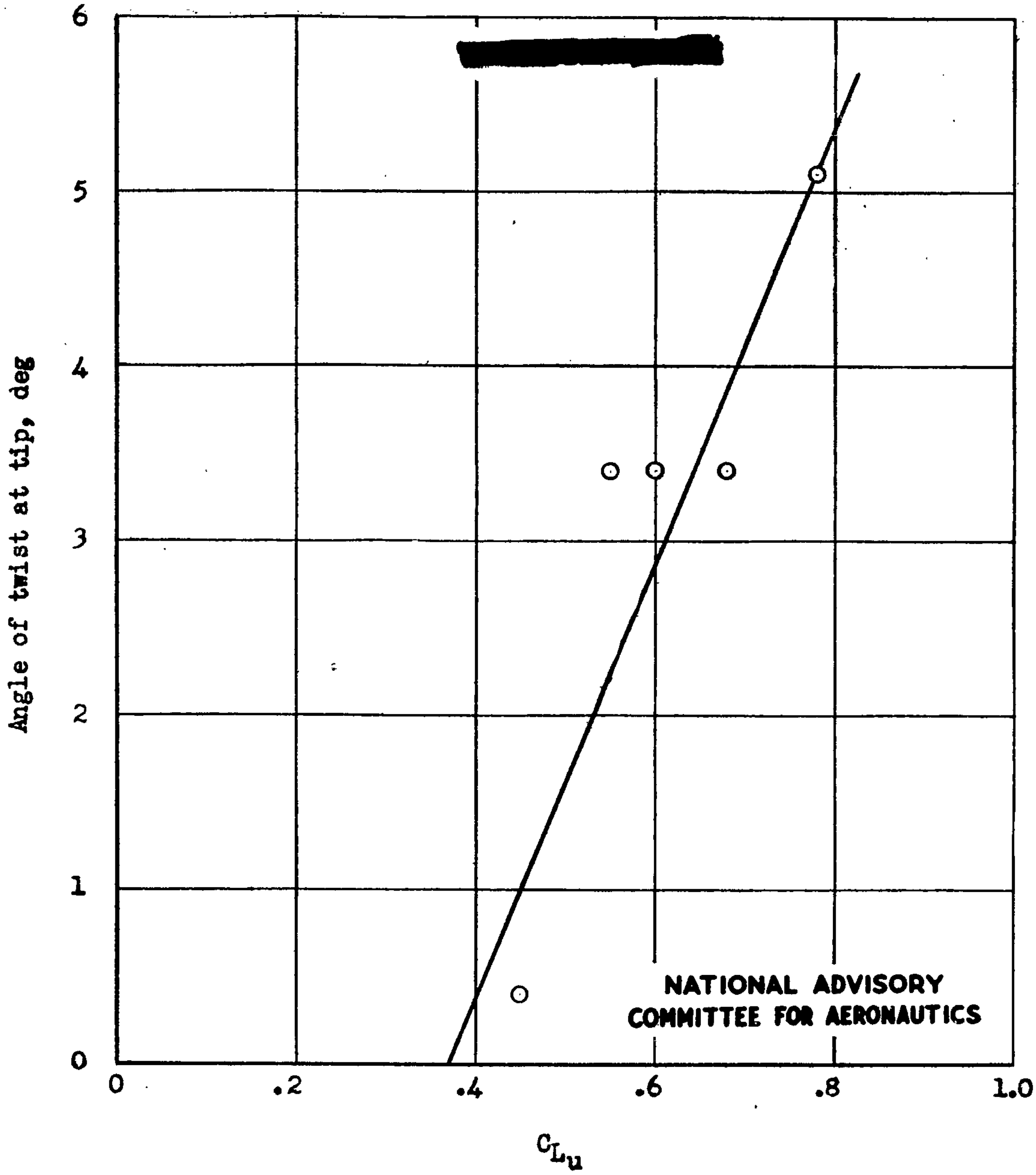


Figure 7.- Twist of propeller tip as function of C_{L_u} for propeller A. Tip speed constant at $q/q_{cr} = 0.37$.

[redacted]